Kinematics and Discontinuous Deformation Analysis of Landslide Movement

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ABSTRACT: Kinematics plays a very important role in the behavior of landslides in discontinuous media. Discontinuous Deformation Analysis (DDA) is introduced as a technique that allows a realistic incorporation of kinematics in routine analyses. The advantage of using DDA lies in the fact that the actual mode of failure does not have to be assumed *a priori*, and displacements and velocities are computed as an integral part of the analysis. Analysis of the Vaiont landslide is used to illustrate the importance of kinematics in the behavior of large landslides to demonstrate the accuracy and capability of DDA.

RESUMO: La cinemática juega un papel muy importante en el comportamiento de deslizamientosde tierra en medios discontinuous. El "Análisis de Deformación Discontinua" (DDA) se introduce como una técnica que permite la incorporación realista de la cinemática en análisis de rutina. La ventaja de usar DDA reside en el hecho que el modo de rotura real no se ha de asumir *a priori*, y tanto desplazamientos como velocidades son calculados como parte integral del análisis. El análisis del deslizamiento de Vaiont se usa para ilustrar la importancia de la cinemática en el comportamiento de grandes despredimientos de tierras, y demostrar así la exactitud y capacidades de DDA.

1. INTRODUCTION

Analysis of slope stability is one of the classic problems in geotechnical engineering. The traditional approach is to use limit equilibrium methods to evaluate the factor of safety against failure (Duncan, 1996). In general, these methods consider only force and moment equilibria, and deformations are not accounted for. Thus, they are well suited for the analysis of the potential failure initiation, but they are poorly suited for the evaluation of actual displacements. Also, correct determination of the mode of failure is essential to their successful application.

More recently, continuum methods such as the finite element method (Zienkiewicz, 1971 and 1977; Desai and Abel, 1972; Zienkiewicz and Taylor, 1989 and 1991) have been applied to the analysis of slope stability. Continuum methods are not limited to simple computation of force and moment equilibria, and can be used to assess deformations. Also, a specific mode of failure, i.e. failure surface location and geometry, does not have to be assumed. However, these methods are generally limited to the analysis of relatively small displacements.

In contrast, the behavior of discontinuous media, such as jointed rock masses is governed by discrete displacements along specific discontinuities. Limit equilibrium solutions that allow the assessment of the potential for failure initiation are available for quite a few simple situations, plane and wedge sliding, and toppling, for instance (Hoek and Bray, 1981). Kinematic methods, which consider the influence of block geometry on the mode of failure, are also available. These include stereographic projection techniques, which are commonly used to evaluate the stability of rock slopes (Goodman, 1976), and block theory (Goodman and Shi, 1985), which may also be employed for this purpose. In the finite element context, various procedures have been employed to account for the effects of discontinuities (Duncan and Goodman, 1968; Goodman et al., 1968; Wang and Voight, 1970; Ghaboussi et al., 1973). However, none of these methods are really intended for modeling the behavior of a system consisting of a large number of individually deformable blocks. This is the realm of the discrete numerical methods, the distinct element method (DEM) and discontinuous deformation analysis (DDA). The strength of discrete methods is their ability to capture the dynamics, kinematics, and deformability of a large number of individual blocks. DEM is a force-based method developed in the early 1970's (Cundall, 1971 and 1987). DDA is a displacement-based method developed during the 1980's (Shi and Goodman, 1984; Shi, 1988). Both have been applied successfully to analysis of slope stability problems.

The purpose of this paper is to introduce DDA as a tool for slope stability analysis of discontinuous rock masses and to explore the role of kinematics in the behavior of landslides.

2. DDA AS A SLOPE ANALYSIS TOOL

DDA has been successfully applied to a wide variety of geotechnical problems, from tunneling to toppling (Yeung, 1991). The more influence the individual material discontinuities have, the more appropriate the DDA model. The importance of the discontinuities is dictated by the scale of the problem.

DDA is a fully dynamic numerical method that models the development of displacements with time. Consequently, it can be used to predict the initiation of failure and to study the subsequent kinematics of the failed mass. The mode of failure is one of the results of the analysis, rather than one of the underlying assumptions. In addition, DDA reproduces the actual displacements along discontinuities within the sliding mass, as well as along the failure surface.

DDA is not limited to analysis of initiation of failure, and may be used to study the behavior of the slope after the onset of motion. This is one of the major strengths of the method.

2.1 *Theory of the DDA Method*

DDA models a discontinuous material as a system of individually deformable blocks that move independently without interpenetration (Shi, 1988 and 1993). Its formulation is based on a dynamic equilibrium that considers the kinematics of individual blocks as well as friction along the block interfaces. The displacements and deformations of the blocks are the result of the accumulation of a number of small increments, corresponding to small time steps. The transient formulation of the problem, which is based on minimization of potential energy, makes it possible to investigate the progression of block movements with time.

The mechanical interactions of the blocks and their surroundings are formulated in terms of the displacement parameter set $\underline{\mathbf{D}}$. These interactions include various loads, block inertia and elastic deformability, and displacement constraints due to block contacts and boundary conditions. The minimum energy solution is found by setting the partial derivatives of the total potential energy function, the sum of the individual energy contributions, equal to zero. This results in a system of linear equations that can be written in matrix form as [K][D] = [F]. For a system of *n* blocks:

$$\begin{bmatrix} K_{11} & \cdots & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & \cdots & \cdots & K_{nn} \end{bmatrix} \cdot \begin{bmatrix} \underline{D}_1 \\ \underline{D}_2 \\ \vdots \\ \underline{D}_n \end{bmatrix} = \begin{bmatrix} \underline{F}_1 \\ \underline{F}_2 \\ \vdots \\ \underline{F}_n \end{bmatrix}$$

A complete description of the formulation of the equations, including derivations of each of the energy terms, can be found in Shi (1990 and 1993) and Shi and Goodman (1984 and 1985).

The original first-order formulation of DDA incorporates a six-member displacement parameter set $\underline{\mathbf{D}} = [u_0, v_0, \theta, \varepsilon_x, \varepsilon_y, \gamma_{xy}]$ for each block that consists of the x- and y-translations (u_0, v_0), rotation (θ), x- and y-components of normal strain (ε_x, ε_y), and shear strain (γ_{xy}). These parameters describe the displacement of the center of mass of the block, from which the corresponding locations of the block vertices can be determined through the use of a first-order displacement function:

$$u = u_0 - (y - y_0)\,\theta + (x - x_0)\cdot\varepsilon_x + \frac{(y - y_0)}{2}\cdot\gamma_{xy}$$

$$v = v_0 + (x - x_0)\,\theta + (y - y_0)\cdot\varepsilon_y + \frac{(x - x_0)}{2}\cdot\gamma_{xy}$$

where (u, v) are the displacements of a point located at (x, y) within a block whose centroid is located at (x_0, y_0). This displacement function, while extremely fast and efficient, uses a very simple approximation for the displacements due to rotation of the block. If needed, the error introduced by this simplification can be corrected by incorporating higher order rotation terms (MacLaughlin, 1997).

The kinematic constraints on the system of blocks are imposed using the penalty method, described as follows. Contact detection is performed in order to determine which block vertices are in contact with edges and vertices of other blocks. Numerical penalties analogous to stiff springs are applied at the contacts to prevent interpenetration of the blocks. Tension or penetration at the contacts results in expansion or contraction of these "springs", which adds energy to the block system. Thus, the minimum energy solution is one with no tension or penetration. However, the forces on each block are in equilibrium, so the energy of the contact force between the blocks is balanced by the penetration energy. When the system converges to an equilibrium state, there are very small penetrations at each contact. The energy of the penetrations is then used to calculate the contact forces, which are in turn used to determine the frictional forces along the interfaces between the blocks. Fixed boundary conditions are implemented in a manner consistent with the penalty method formulation.

Solution of the system of equations is iterative: contact springs are repeatedly added and subtracted until each of the contacts converges to a constant state. The positions of the block vertices are then updated according to the prescribed displacement function.

2.2 *Numerical Implementation*

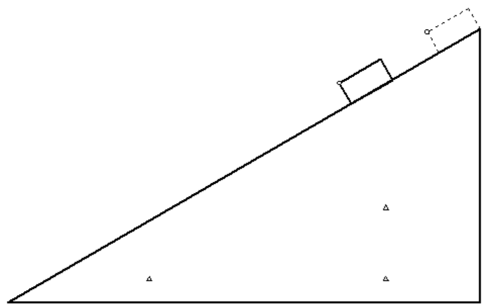
The computer implementation of DDA used in this study is a set of programs originally written by Shi (1993), as adapted for the MS-Windows™ environment by MacLaughlin and Sitar (1995). In this implementation, each discontinuity (joint) may have a different friction angle, cohesion, and tensile strength. The blocks are deformable, with constant stress and strain within each block governed by the Young's modulus and Poisson's ratio.

2.3 *Numerical Accuracy*

Yeung (1991) and MacLaughlin (1997) extensively tested the accuracy of the DDA code. Their principal effort concentrated on testing of static and dynamic response using a variety of geometries and failure modes. Here, a simple example used by MacLaughlin (1997) is shown to illustrate the ability of the method to accurately predict large displacements and velocities.

The problem consists of a single block sliding down an inclined plane as depicted in Figure 1a. When the friction angle is less than

a)



b)

Figure 1. a. Sliding block on a 30° slope; b. Comparison of DDA and analytical results.

the slope angle, the block will accelerate down the incline. For a block initially at rest under acceleration of gravity *g*, the analytical solution for its displacement *d* as a function of time *t* is:

$$d = \frac{1}{2}at^2 = \frac{1}{2}(g\sin\alpha - g\cos\alpha\tan\phi)t^2$$

Figure 1b shows a plot of the analytical solution and the DDA results for a 30° slope with three different values of ϕ (0°, 10°, and 20°). The results compare very well and the displacements computed using DDA are within 1% of the theoretical solution.

3. INFLUENCE OF GEOMETRY AND KINEMATICS

The importance of considering kinematics in the analysis of slope stability is illustrated here first by using DDA to model two relatively simple cases: a compound, planar, and a circular-shaped failure surfaces.

3.1 *Compound Failure Surface*

Figure 2 shows a slope with a compound failure surface that is comprised of two inclined planes, at 75° and 15°. For the case of 12 discontinuities, the slope fails by sliding (Figure 2a) unless the friction angle along the vertical discontinuities is low enough to allow toppling to occur (Figure 2b). Kinematic analysis also gives an indication of the variation of the displacements across the slope, due to differential movement between the blocks. Sliding failure of the slope results in almost the same amount of horizontal displacement at the head scarp and at the toe. However, the failure plane under the scarp is much steeper than under the toe, which means that the corresponding vertical component is much larger at the scarp. Thus, the total displacement near the head scarp is much greater than the total displacement of the toe, as shown in Figure 2.

Toppling failure (Figure 2b) involves forward rotation of the blocks, which accommodates relatively large displacements at the head scarp with small displacements at the toe. Either mode of failure may result in a displacement pattern consisting of a graben at the head scarp with less discernible movement at the toe, which is a fairly common field observation in landslide investigations.

a)

b)

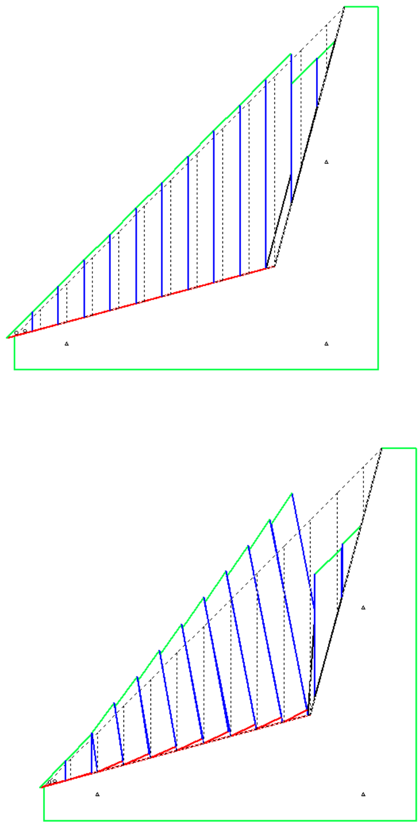
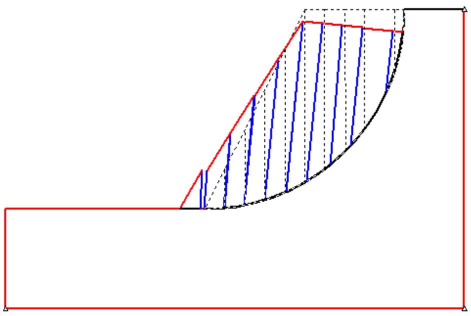


Figure 2. Kinematics of failure along a compound surface: a) pure sliding; b) sliding and toppling.

3.2 *Circular failure surface*

Figure 3a shows a 2:1 (V:H) slope with a circular failure surface, and divided into vertical blocks (slices). If this sliding mass is analyzed as a single block, a friction angle of 38° is required to prevent failure. However, toppling becomes a possible mode of failure when the friction angle between the vertical slices is sufficiently low and the friction angle on the failure surface is high, as shown in Figure 3b. In this case, the friction angle along the failure surface, which is required to maintain stability, is higher than that required to prevent a rotational failure. This alternate mode of failure would not be identified using a traditional limit equilibrium analysis. Consequently, the friction required for stability might be severely underestimated and the factor of safety overestimated.

a)



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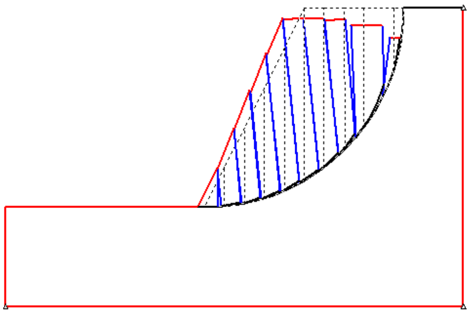


Figure 3. Kinematics of failure along a circular failure surface: a) rotation; b) translation and toppling.

4. VAIONT LANDSLIDE

We chose the Vaiont landslide, which occurred in northern Italy in October 1963, to investigate the role of kinematics in controlling the movement of large landslides. The slide took place during the filling of the reservoir formed by the construction of a large concrete arch dam in the Vaiont River valley. An enormous amount of rock, on the order of 200-300 million cubic meters, suddenly broke loose and slid down into and across the reservoir, displacing the water over the dam and causing substantial loss of life downstream (Jaeger, 1979, Voight, 1979).

The rock mass, predominantly thick beds of limestone separated by layers of clay, had been moving slowly for several years prior to the catastrophic failure. A cross-section of the pre-slide geometry is shown in Figure 4. The movement rates appeared to be highly correlated with reservoir level and had been observed for some time (Muller, 1964). The cumulative displacement along the failure plane prior to the main failure was on the order of several meters or more (Muller, 1968) before the main slide event.

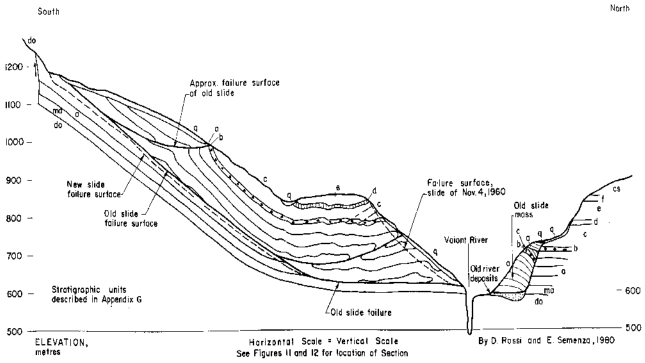


Figure 4. Gelogic section of Vaiont slide (reproduced from Herndon and Patton, 1985).

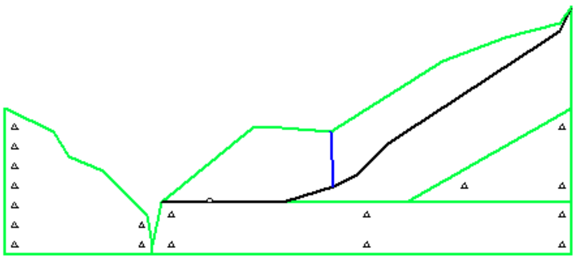
A large number of two-dimensional limit equilibrium analyses using methods of slices were performed after the failure by various investigators. The friction angles required for stability back-calculated from these analyses range from $\phi = 17.5°$ to $\phi = 28°$. However, strength test data on the clay material along the failure surface show friction angles ranging from 5° to 16°, with an average value around 12° (Hendron and Patton, 1985). With $\phi_{available}$ along the failure surface < $\phi_{required}$ for stability, the slope should not have been stable even before the filling of the reservoir. Since the slope had been at least marginally stable for quite some time prior to failure, it may be concluded that there are factors controlling the stability that are not accounted for in the two-dimensional limit equilibrium analyses.

Hendron and Patton (1985) were able to resolve this discrepancy through the use of three-dimensional stability calculations. Sensitivity analysis performed as part of their extensive two-dimensional stability studies indicate that changes in the groundwater and reservoir levels could have affected the factor of safety of the slope by as much as 10 to 15%. The value of the interslice friction angle could change the factor of safety by approximately the same amount. In addition, the high velocity of the landslide was attributed to pore pressures generated by boiling water along the failure surface.

Lo et al. (1971) conducted a limit equilibrium analysis of the Vaiont slide, using Janbu's method for noncircular surfaces. Their model of the geometry of the slide consisted of two wedges separated by a vertical discontinuity located near the center of the slide mass. For groundwater level corresponding to the height of the water in the reservoir, they calculated $\phi_{required}$ = 13°.

In our DDA analyses we used a simplified cross section of Hendron and Patton (1985) which we subdivided into different number of blocks (Figure 5). In the simplest case we assumed that the mass behaved as a single block. The results indicate that under completely dry conditions, a slide mass represented by a single block would require a friction angle of only 8° for stability. However, if the mass is divided by a single vertical discontinuity into two blocks (Figure 5a) the required friction angle along the slide plane varies between 8° and 14°, depending on the interblock friction and the position of the vertical discontinuity (MacLaughlin, 1997). The DDA result for a configuration very similar to that used by Lo et al. (1971) was 14°, which compares very well with their result.

a)



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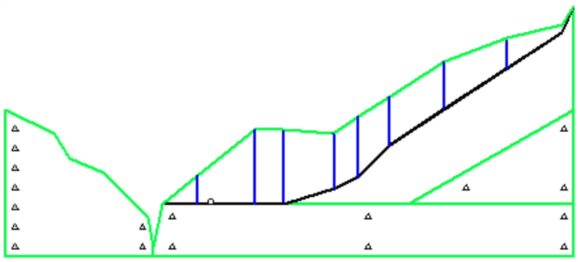

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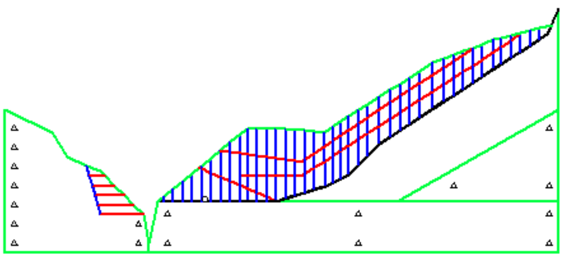

Figure 5. Simplified section through the Vaiont slide used in DDA: a) 2 blocks; b) 9 blocks; c) 105 blocks.

The position of the vertical discontinuity between the blocks also significantly affects the results and computed $\phi_{required}$ varies from 8° to 14° (MacLaughlin, 1997). The lowest values of $\phi_{required}$ correspond to a vertical discontinuity located near the toe and crest of the slide where the resisting/driving force ratio is highest. The highest values correspond to a vertical discontinuity located near the middle of the slide (near the break in the slope of the failure surface), where the resisting/driving force ratio is minimized. Chowdhury, in an analysis of the Vaiont slide using a limit equilibrium method that models progressive failure, produced results with an almost identical trend (Chowdhury, 1978).

The 9-block sliding mass in Figure 5b corresponds to one of the configurations used in Hendron and Patton's two-dimensional limit equilibrium analyses. The value of limiting friction angle required for stability computed using DDA, $\phi_{required}$ = 15°, compares very well with $\phi_{required}$ = 14.9° calculated by Hendron and Patton. Additional DDA analyses indicate that the $\phi_{required}$ increases with increasing number of slices, particularly when the slices are concentrated near the middle of the slide mass, near the change in the slope of the failure surface. The results are summarized in Table 1. Eventually, there are enough discontinuities concentrated around the slope break that most of the kinematic constraints have effectively been removed and adding more discontinuities does not affect the stability of the slope. The $\phi_{required}$ for stability at this point corresponds to that calculated with the limit equilibrium methods of slices. However, the computed velocity of the slide mass is very much afftected by the number of blocks, as will be discussed next.

Table 1. $\phi_{required}$ as a function of the number of blocks in DDA.

Number of blocks	$\phi_{required}$
1	7°
2	13°
3	15°
9	18°
23	15°
105	16°

The behavior observed in the DDA analyses is consistent with the "discontinuous flow" model proposed by Jaeger (1968). It is essentially an extension of the progressive failure concept, with the unstable upper portion of the slide creeping gradually down slope, imparting greater and greater forces on the lower stable portion of the slide. Eventually, the forces increase to the point where they are high enough to cause sudden rupture within the lower stable zone. Jaeger cites evidence of a nonuniform zone of deterioration and physical weakening which separated the upper sliding mass from the lower. Borehole displacement data in fact indicate that the upper block was indeed moving as a more or less intact block of material. The manifestation of the instability of the upper block was a continuous downslope creep, with a total cumulative displacement of 0.5 m to 2.5 m. The seismic velocities of the material, originally around 6000 m/s, a typical value for intact rock, had dropped to less than 3000 m/s in a very short time, indicating an increasing level of fracturing within the lower mass, most likely associated with some dilation. Seismic records also provide evidence that a brittle fracture occurred within the rock mass immediately prior to the main slide event. Thus, there is a considerable amount of evidence indicating that the discontinuous flow model of the behavior of the slide is plausible. Furthermore, if internal deformation and fracturing resulted in formation of major discontinuities in the middle of the sliding mass, the slide would have broken up into multiple pieces. This scenario is consistent with typical behavior of large landslides.

A very interesting aspect of the Vaiont slide is the extremely high rate of movement. The sliding mass moved a total of over 400 m (including a relatively large upward vertical component of displacement of 150 m) in less than 60 seconds. Previously published estimates of maximum velocity, based on a variety of observations ranging from measurements of the final configuration of the sliding mass to distance traveled versus duration to wave heights in the reservoir, range from 20 to 50 m/s (Hendron and Patton, 1985). These values are much higher than had been anticipated. Velocity estimates prior to failure, based on physical model studies as well as experience with other slides having similar geometries, were an order of magnitude lower. Various hypotheses have been developed to explain the anomalously high velocities, including reduction of shear strength on the failure surface and heat-generated pore pressure.

Since DDA is a fully dynamic method, it can be used to calculate the velocity of the sliding mass as a function of time, as well as the maximum velocity achieved during the slide. We analyzed the different block configurations in order to quantify the effect of block size on the velocity calculations, including the 9-block configuration of Hendron and Patton (1985) shown in Figure 5b. In the DDA models, the friction angle for dry conditions on the failure surface was assumed to be $\phi_{available} = 12°$. The results obtained from the different configuration by DDA are presented in Figure 6 and are compared to an analytical solution, which accounts for strength loss due to pore pressure generation caused by frictional heating of the sliding masses (Hendron and Patton, 1985). As can be seen, the maximum velocity achieved increases with the number of blocks. The closest agreement between the analytical solution and DDA is obtained with a model consisting of 105 blocks (Figure 5c).

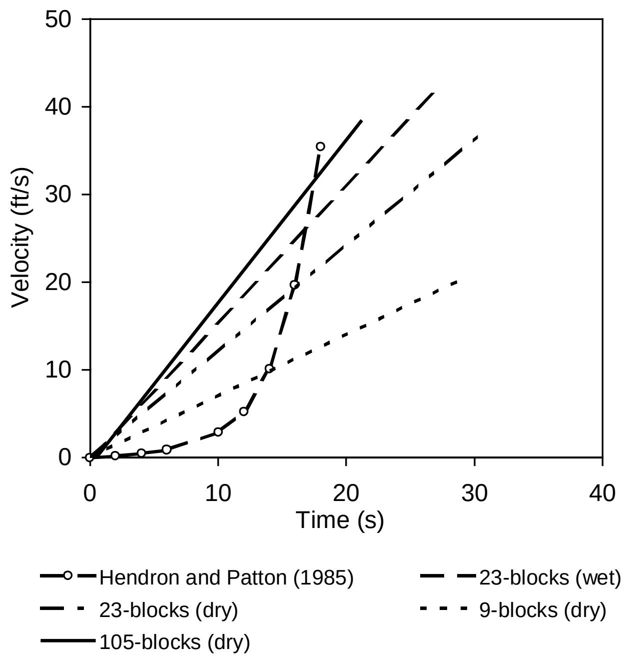

Figure 6. Velocity as a function of time for different block configurations as compared to an analytical solution.

The most significant difference in the results is in the shape of the curves, since the DDA code currently does not account for any changes in strength with displacement, resulting in constant acceleration. In contrast, the model used by Hendron and Patton (1985) does incorporate strength loss and displays increasing acceleration. Overall, however, the results show that kinematics has a very significant influence on the predicted velocity of sliding. In the case of the Vaiont example the influence of kinematics tends to overshadow the influence of variations in shear strength along the failure plane. This result suggests, that investigations of slides in blocky materials should be place as much emphasis on the investigation of the geometry of the landslide blocks and their interaction, as is spent on the determination of shear strength characteristics.

5. CONCLUSIONS

Kinematics plays a very important role in the behavior of landslides in discontinuous media. New techniques, such as DDA, allow a realistic incorporation of kinematics in routine analyses. The advantage of using DDA lies in the fact that the actual mode of failure does not have to be assumed *a priori*. As shown using the Vaiont landslide example, the method is accurate, and it also allows the computation of displacements and velocities, which are not easily obtained using traditional limit equilibrium methods.

6. ACKNOWLEDGMENT

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